

# Approximating the irrational number $e$

$$\begin{aligned}e_1 &= \frac{1}{0!} + \frac{1}{1!} = 1 + 1 = 2 \\e_2 &= e_1 + \frac{1}{2!} = e_1 + \frac{1}{2} = 2.5 \\e_3 &= e_2 + \frac{1}{3!} = e_2 + \frac{1}{6} = 2.\bar{6} \\e_4 &= e_3 + \frac{1}{4!} = e_3 + \frac{1}{24} = 2.708\bar{3} \\e_5 &= e_4 + \frac{1}{5!} = e_4 + \frac{1}{120} = 2.71\bar{6} \\e_6 &= e_5 + \frac{1}{6!} = e_5 + \frac{1}{720} = 2.7180\bar{5} \\e_7 &= e_6 + \frac{1}{7!} = e_6 + \frac{1}{5040} = 2.71825396\bar{8} \\e_8 &= e_7 + \frac{1}{8!} = e_7 + \frac{1}{40320} = 2.718278769841\bar{2} \\e_9 &= e_8 + \frac{1}{9!} = e_8 + \frac{1}{362880} = 2.718281525573192239858906 \\e_{10} &= e_9 + \frac{1}{10!} = e_9 + \frac{1}{3628800} = 2.71828180114638447971781305 \\e_{11} &= e_{10} + \frac{1}{11!} = e_{10} + \frac{1}{39916800} = 2.7182818261984928651595318 \\e_{12} &= e_{11} + \frac{1}{12!} = e_{11} + \frac{1}{479001600} = 2.7182818282861685639463417241195018972796750574528352306130083907 \\e_{13} &= e_{12} + \frac{1}{13!} = e_{12} + \frac{1}{6227020800} = 2.718281828446759002314557870113425668981224536780092335647891203 \\e_{14} &= e_{13} + \frac{1}{14!} = e_{13} + \frac{1}{87178291200} = 2.71828182845822974791228759482727736695990664244632498600752569006 \\ & \qquad \qquad \qquad 537260505514473768442022410276378530346784315038283292 \\ & \qquad \qquad \qquad 251546219800188054156308124562092816061070029323997577 \\ & \qquad \qquad \qquad 965831934085902339870593838847807101775355743609711863 \\ & \qquad \qquad \qquad 680117648371616625584879553133521387489641457895426149 \\ & \qquad \qquad \qquad 394403362657330911299165267419235673203927172181140435 \\ & \qquad \qquad \qquad 108689076943045197013450981704949958918212886466854720\end{aligned}$$